Chapter 3

Unifying Opposites

When all the world recognizes beauty as beauty, this in itself is ugliness.
When all the world recognizes good as good, this in itself is evil.

Lao Tzu

In the previous chapter, I described the first part of the data model of the Universe and the semantic system of coordinates that arises from it. I now come to the second, and far more important part of the data model, the Principle of Duality, which will evolve into the Principle of Unity in the next chapter, when we include the Divine in IRL. It is the Principle of Duality, as applied to the relativistic world of form, that provides us with the underlying principle for a synthesis of everything. Indeed, the Principle of Duality even underlies the description of the unified structure of the Universe, as described on page 217 in Chapter 2, 'Building Relationships'.

It is at this stage that the importance of clearing the mind of all conditioned learning is most important, for this concept cannot be understood with any system of thought that has been inherited from Aristotelian logic. A completely different way of looking at the Universe is required from the methods that dominate Western culture today, changing from a predominately either-or approach to an all-inclusive both-and one.

We are entering what for many is a very strange, absurd world, wonderfully depicted in Lewis Carroll's *Through the Looking Glass and what Alice found there*. For Carroll—as Charles Dodgson, an Oxford mathematical don—knew a thing or two about logic (he wrote a couple of books on the subject),[1] and the existence of a world that defies the laws of logic as they have

been assumed for millennia. To heal the fragmented mind in Wholeness, we need to pass through a looking-glass and enter the world of paradoxes, like Alice.[2]

However, the looking glass that Alice passed through is not an ordinary mirror; it is more like a two-way mirror, enabling us to see both the Universe as a whole without restrictions and the interior of the room, which represents specialist, fragmented knowledge. But people in the room can only see their own reflections, unaware that there is a weird and wonderful world outside. In this liberating, awakening, and healing manner, we can escape from Plato's cave of illusionary shadows[3] and discover not so much Plato's eternal Forms and Ideas,[4] which correspond to ever-changing classes in IRL, but Nondual, Formless Wholeness, which alone is Immortal Reality, as we see in the next chapter.

However, we do not abandon the fundamental egalitarian process of forming concepts on the other side of the mirror; we continue to look at the similarities and differences in the data patterns of our experiences, just as described on page 181 in Chapter 2, 'Building Relationships'.

As IRL is noninferential and has no assumed truths, it is not constrained by the orthodox mathematical need for axiomatic consistency. So we can continue to use the commonsensical principle of consistency, described on page 141 in Subsection 'Principles of conceptual modelling' in Chapter 1, 'Starting Afresh at the Very Beginning', quite different from that which prevails in deductive systems of mathematical proof and inferential logic. For, as is well known, if the axioms of mathematics are inconsistent, then it is possible to prove both a theorem and its opposite from the axioms. Conversely, if it can be proved that a theorem and its opposite cannot be proved from the axioms, such a proof would constitute a proof of the consistency of mathematics.

Such a proof is possible for the propositional calculus, as Nagel and Newman show, for instance.[5] This is perhaps not surprising because the propositional calculus is concerned only with tautologies. This branch of conventional logic is therefore hardly likely to lead us closer to the Truth. However, as Kurt Gödel showed in his Incompleteness Theorems of 1931, it is not possible to prove that the axioms of arithmetic are either consistent or complete using the deductive logic of metamathematics.[6] As Gödel's theorem also applies to the axioms of mathematical logic, it is not even possible to prove that the axioms of deductive logic are consistent.

To be able to represent the concepts of Gödel's incompleteness theorems in IRL, and also a multitude of other phenomenon that cannot satisfactorily be expressed in conventional Western systems of thought, another approach to opposites is required. That, in essence, is what the Principle of Duality provides.

The Principle of Duality

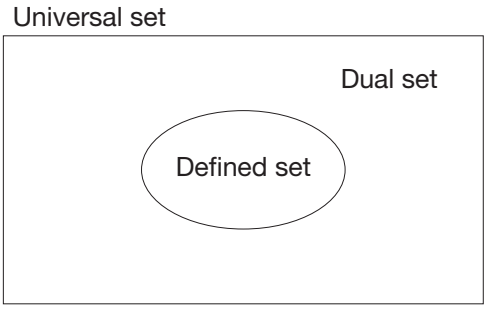


Figure 3.1: *Dual sets within the universal set*

To understand something of the nature of the Principle of Duality, we must remember that the way that we form concepts is to look for the similarities and the differences in the data patterns or percepts of our experience. Similar percepts are collected together in one set and different percepts are put into other sets. However, there is one specific type of difference that is different from differences in general. Whenever we form a set of percepts, we do so in the context of the universal set, which represents the context for the concept being formed. This process divides the world into two, which we can call the defined set and its dual, as illustrated in Figure 3.1.

In other words, whenever we form a concept, we also form its opposite. For instance, if we consider the universal set of all arithmetical statements, then we can define a class of entities called 'mathematical fact' that has an attribute with the domain of values 'true' and 'false'. Thus '2+2=4' is an entity in IRL with an attribute value 'true' and '2+2=5' has an attribute value 'false'. These two entities have some attributes in common and so could be put into one set. For example, syntactically they are both well-formed arithmetic statements. However, they have one property, the value of the attribute 'truth value', which is not only different, it is contradictory. Accordingly, we can put all arithmetic statements with attribute values 'true' and 'false' into two different sets. These we can call dual sets.

Notice here that we do not reject false statements from IRL. If the URT is to include everything, it must include both true and false statements. It is only by doing this that all dualities can be transcended so that Nondual, Ineffable Truth can be realized. So the URT includes both a geocentric and a heliocentric view of the solar system and both an anthropocentric and theocentric view of the Universe.

This is but one example of dual sets, but there are many others. In general we can say that a set B is the dual of A if it has an attribute value that is the opposite or pole of A. It follows from this that set A is also the dual of B. Duality or polarity is a universal feature of the Universe as it must be because every time we form a set we automatically form its dual.

So just as there can be no truth without falsity, there can be no positive without negative, no dark without light, no good without bad, no life without death, no attraction without repulsion, no happiness without sadness, no rationality without intuition, no male without female, and no yin without yang. It is not always immediately obvious that attribute values are duals of each other. Sometimes they have this relationship simply by the way that the notation

is being used. For example, '0' and '1' represent dual concepts in the theory of probability and in computer systems, but in many other contexts they do not have this relationship.

Neither is duality always intuitively obvious, as we can see from an example from projective geometry. First, Blaise Pascal discovered in 1639, when he was sixteen years old, that if six points are placed on a conic section and joined as in Figure 3.2, then their points of intersection, *LMN*, are collinear. Because straight lines remain straight lines in conical projections, this property applies not only to the ellipse, as in the diagram, but also to the parabola and even hyperbola, consisting of two disconnected open curves. As such a property is not intuitively obvious, it is not surprising that Pascal called the six points *ABCDEF* his Mystic Hexagram.[7]

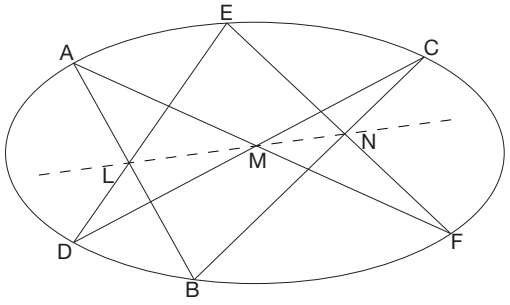


Figure 3.2: *Pascal's Mystic Hexagram*

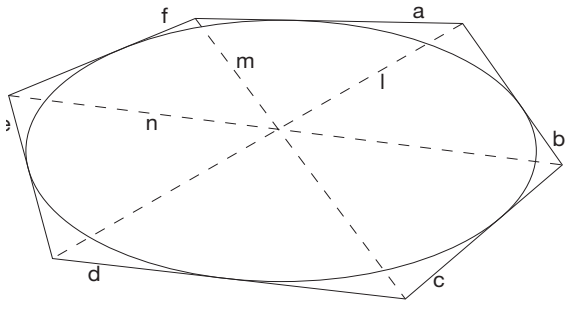


Figure 3.3: *Brianchon's dual theorem*

Nearly two hundred years later, in 1810, Charles Julien Brianchon proved a related theorem, illustrated in Figure 3.3. If six lines are drawn tangentially to a conic section to form a hexagon, as *abcedf*, then the lines joining opposite vertices, *lmn*, intersect at a single point.[8] The relationship between these two theorems can best be seen from an observation made by Florimond de Beaune, a friend and student of René Descartes in the seventeenth century: a curve may be regarded both as the path of a moving point and as the envelope of a moving line.[9]

Pascal and Brianchon's theorems are examples of what is called the Principle of Duality in projective geometry, which is where the Principle of Duality in IRL comes from. Whatever theorem can be proved about points and lines has a dual or reciprocal theorem about lines and points, where lines and points are interchanged. This leads to the interesting possibility that if a theorem about lines and points is difficult to prove, it is sometimes easier to invert the diagram into points and lines and to prove this dual theorem, a possibility that fascinated me as a mathematics undergraduate in the early 1960s. For proving one theorem proves its dual.

Of course, the Principle of Duality applies not only in two dimensions. For instance, the tetrahedron is self-dual, illustrated in Figure 1.17 on page 85, and the great stellated dodec-

ahedron, discovered by Johannes Kepler in 1619, and the great icosahedron, discovered by Louis Poinsot in 1810, are duals of each other.[10]

Not unnaturally, we can put all these dual concepts into a relation called 'Dualities'. It is a very large relation, part of which is illustrated in Table 3.1. Notice that each entry in the relation includes the particular context for each pair of concepts and that it does not matter which column the two concepts are put into. The domain of values for columns Arbitrary A and Arbitrary B have nothing in common. The attribute values in the two columns could be reversed for any particular row without any distortion in meaning.

These examples are all general concepts or universals. But in IRL, we also regard specific concepts or particulars as concepts. Thus any particular situation can also be viewed in opposite ways; there are two sides to every story. If both these perspectives are not looked at at the same time, the result is a conflict of opposites and a lack of Wholeness.

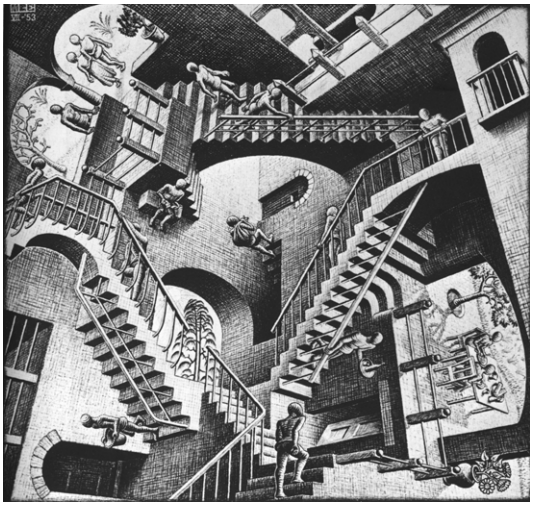


Figure 3.4: *Relativity*

Of course, this conflict is often most uncomfortable, leading to dis-ease and a lack of health. Escher's lithograph *Relativity*[11] well depicts the discomfort that we can feel when confronted by the conflict of opposites. For in a world in which everything is relative to everything else, situations can often appear in quite contradictory ways, depending on which way we look at them.

So what most of us do in our Western culture is to accept one side of the duality and reject the other; we do not follow Fritz Schumacher's maxim to "Accept everything; reject nothing".[12] Being unable to handle this conflict within ourselves, we project it out on to the world, the result being the disharmony and unhappiness that we see in the world today.

We can encapsulate all these dual relationships into a single statement, called the Principle of Duality (D):[13]

A complete conceptual model of the Universe consists entirely of dual sets.

Fairly obviously, if D is to form the central concept of IRL, we must ascertain whether this statement has the attribute value 'true' or 'false'. The answer is that it has both. Many dual sets exist, which while not verifying D, do not invalidate it. On the other hand it is very easy

Class name	*Dualities*		
Attribute name	*Context*	*Arbitrary A*	*Arbitrary B*
Attribute values	noticing	similarity	difference
	representing relationships	entity	attribute
	parts of statements	subject	predicate
	totality of existence	absolute	relative
	data patterns	whole	part
	scientific philosophy	holism	reductionism
	proposition	universal	particular
	passive interpreted data types	knowledge	information
	general data types	active	passive
	data reference	name	value
	business modelling	process	entity
	structural relationships	hierarchy	network
	reproductive partners	male	female
	universal principles	yin	yang
	approaches to life	East	West
	extremes of colour	black	white
	value judgement	good	bad
	linear time	past	future
	dimensions of time	now	past-future
	existential	form	formless
	theories of physics	relativity	quantum
	properties of relativity and quantum theories	locality, causality, continuity	nonlocality, noncausality, noncontinuity
	attitude	profound	superficial
	geometrical duality	line	point
	opinion	like	dislike
	state of being	happy	unhappy
	meaningful statements	true	false
	philosophy	certain	uncertain
	axiomatic logic	consistent	inconsistent
	general	A	not-A

Table 3.1: *Class of dualities*

to falsify D. The basic definition of set is that it is a collection of entities that have a common property. So a collection of entities that do not have a common property do not constitute a set. Thus D is false. But is it? Collections of entities with no common property have the com-

mon property that they have no common property. So we can say that a collection of entities with no common property is in the set of all collections of entities with no common property, which is the dual of the set of all sets by the definition of set. A collection of entities that are not in sets is often called *miscellaneous*.

A special type of miscellaneous set has been a source of much conflict among mathematicians.[14] This is the set formed by the axiom of choice. The axiom of choice states "given any collection of sets, finite or infinite, one can select one object from each set to form a new set". For example, a set could be formed by selecting one person at random from each country in the world. Because of the arbitrary way that this set is formed, there is no readily identifiable common property among the members that makes it obvious that this set is actually a set. The only common property that the members have is that each member has been drawn from a different set from all the other sets. It is actually a set that is formed in exactly opposite way from normal sets. We could call it a non-set set.

Feeling uneasy about the definition of such a set, a number of mathematicians are unwilling to accept the axiom of choice and have developed a branch of mathematics in which the axiom of choice is not assumed to be true. On the other hand, the axiom of choice is needed to prove many theorems in mathematics. This situation can quite easily be modelled by the Principle of Duality. For we can put all the theorems that are deducible from a set of axioms including the axiom of choice in the dual set of all theorems that are derived from axioms that do not include this axiom.

Explaining the logic of the Principle of Duality a little further, another way of looking at this principle is to consider it as a concept in the model. As a concept is defined to be a set of situations or circumstances, we can say that for any set of circumstances in which D is true there is always a set of circumstances in which it is false. Thus D is a self-referencing statement that asserts its own falsity. D can only be true if it is also false.

However, we are now talking at two levels of truth. At one level, D is both true and false, which means that at a higher level D is true. This process can continue indefinitely. At the higher level at which D is true there is also a set of circumstances in which D is false, because that is what D asserts. This shows that D is true at a third level of truth and so on to infinity, as Figure 3.5 illustrates.

This diagram is the backbone of IRL, the skeleton for the Unified Relationships Theory. It is not very upright, but that is the way that it has turned out. Using Hegel's logic, if the thesis is 'D is true' and the antithesis is 'D is false', then the synthesis is 'D is true'. D is thus an irrefutable universal truth, true in all possible worlds and thus lies in the ontological level of the foundations of IRL, illustrated in Figure 1.51 on page 172. The Principle of Duality is not the Absolute Truth. This lies in the gnostic level of the foundations, which we shall look at in the next chapter, when the Principle of Duality becomes the Principle of Unity:

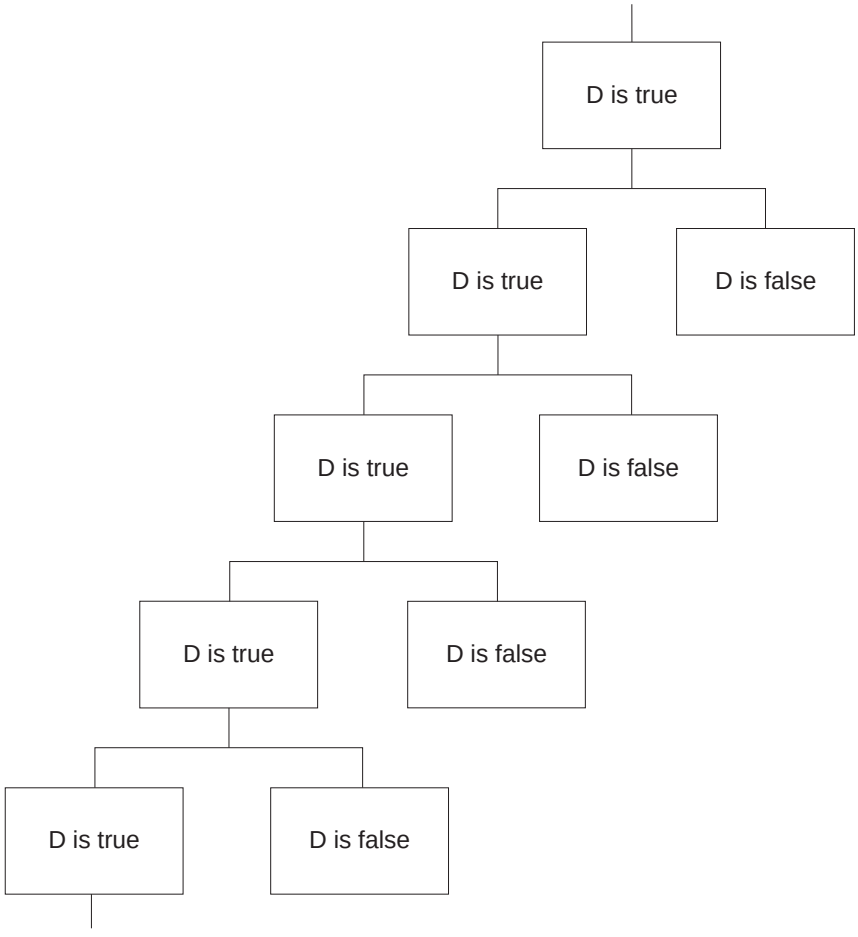


Figure 3.5: *The Principle of Duality*

Wholeness is the union of all opposites., which is also not the Absolute Truth; it is simply a universal truth, the fundamental guiding principle of the Universe.

We can move towards the Truth by noting that the set of circumstances in which D is true and false are not symmetrical; there is a primary-secondary relationship between the statements 'D is true' and 'D is false'. This primary-secondary relationship between dualities is not uncommon in the Universe. Table 3.2 shows just a few of them.

So what happens when two people meet, one of whom is driven by the argumentative mind, the other being free of all inner conflict? Does this situation necessarily lead to conflict? Well, Krishnamurti was faced with just this question at one of his discourses.[15] One of his questioners asserted, "There are two sides to conflict". Krishnamurti replied: "No. There are not two sides. I do not want to have conflict." He went on to say that if his questioner gets angry with him because of this then that is his problem, not Krishnamurti's.

Primary	Secondary
D is true	D is false
Spirit	Mind
Mind	Matter
Intuition	Rationality
Intelligence	Intellect
Feminine	Masculine
Inner knowing	Symbolic knowledge
Eastern cultures	Western cultures
Formless Absolute Whole	Relativistic world of form
Implicate order	Explicate order
Nonduality	Duality

Table 3.2: *Primary-secondary relationships*

We thus live in an upside down world, one that puts the cart before the horse. It is a world that is standing on its head, rather than having its feet firmly placed on the eternal Ground of Being. While the Western world is rational, it is not logical. It is not based on the primary Reality, on the Logos, which arises from our Divine Source. It is because of this that Western civilization is now degenerating at a very rapid rate. But is it too late to reverse this absurdity; to put the horse before the cart where it belongs?

The problem that we face in making this transformation from a dualistic, either-or to a nondualistic, both-and culture is that living completely in a dualistic world is rather like living in a room with no doors, with a wall containing a two-way mirror. Those living outside the room have the freedom to see both the exquisitely beautiful world outside, full of mountains and lakes, trees and running water, and also the dualistic world inside the room. Whereas those imprisoned within the room have no knowledge of any world outside; all they can see is their own reflections, as mentioned on page 224.

While dualities can only truly be reconciled through the experience of a nondual state of consciousness, a partial reconciliation is nevertheless possible within the relativistic world of form. For example, the Principle of Duality could be used to represent the way that arbitrators resolve conflicts between competing groups. The key point is to find some common ground or purpose on which the combatants can agree. If this can be done, a broader context is established enabling the competitors to see that their conflict is not in their mutual interests. This process can take many steps, for as soon as an apparent common ground is found, it is not uncommon for someone to raise an objection to the agreement, and the search for common ground must continue up the chain. However, it should be noted within the world of form, this process of negotiation can never be complete.

Every set of circumstances in the world of everyday affairs contains the potential for conflict within it. By sharing our lives with other human beings, a host of conflicts arise every day

as the newspapers and television screens show us only too clearly. Even such simple things as "what shall we eat today and when shall we eat?" or "where shall we go tomorrow?" can give rise to arguments if the individuals involved are not prepared to give and take. But such arguments can be an important learning experience; we can use them to bring us ever closer to God, to Nondual Wholeness.

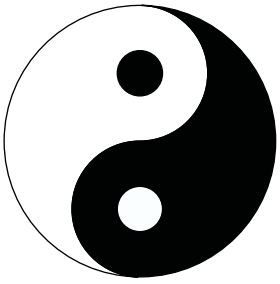


Figure 3.6: *T'ai-chi-t'u*

While the Principle of Duality cannot be accommodated within the principles that underlie Western culture, it is not without precedents. The most obvious of these is the Chinese Tao, as this classic *T'ai-chi-t'u* symbol, or 'Diagram of the Supreme Ultimate', indicates quite clearly. This symbol depicts the cyclic nature of the Universe. For example, day turns into night, which then turns back to day. The dots in the middle of the two main shapes indicate the potential of the opposite to arise when one side is dominant in any particular situation. The key point here is that when the Universe is viewed as a whole, both opposites exist; to reject one in favour of the other does not lead to Wholeness, peace, and tranquillity.

In the West, Heraclitus has most clearly seen that Wholeness is the union of all opposites as this fragment indicates: "The god: day and night, winter and summer, war and peace, satiety and hunger".[16] However, as I have indicated, Aristotle could not accept Heraclitus's logic, thus sending Western thought further and further away from the Truth and Reality, as Figure 4.13 on page 275 shows.

Another who attempted to break free from Aristotelian logic was Hegel. Because of its universal applicability, we can consider the Principle of Duality as a generality of all dialectic arguments, the most notable being the dialectic of Hegel's logic. But we can see that Marx was mistaken to base his political philosophy on Hegel's dialectic *materialism*. The 'wholeness of Man', which was Marx's great dream,[17] can never come about through political revolution; it can only be realized through an inner transformation that transcends the conflict of all opposites.

One pair of opposites that well illustrates the primary-secondary relationship between opposites is perfection and imperfection. If we seek perfection as an ideal, continuously attempting to remove what we might regard as imperfections, we are ignoring the reality of the world we live in. Perfection is the union of perfection and imperfection, a principle that enables us to accept the world and ourselves, just as it is and we are. Otherwise, tensions arise that take us away from Peace, perfect Peace.

Another pair of opposites that displays the same relationship are beauty and ugliness. If we see only beauty around us and ignore the ugliness, we are denying the reality of the world we live in. So if we are to embrace the Totality of Existence, and so feel the beauty of the life

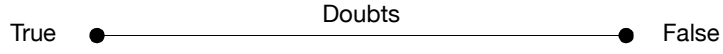
within us, we need to see both sides of this duality. It is only by transcending beauty and ugliness, without judgement, that we can truly see the beauty of the world we live in as the reflection of our own inner beauty.

The circle of duality

So far we have been considering dualities in terms of polarities or opposites. However, the Universe is not solely black and white, as Aristotle apparently believed; it contains many shades of grey. So, it is very clear that Aristotle's Law of Excluded Middle does not hold in the Universe as a whole. Furthermore, neither does Aristotle's Law of Contradiction apply universally, because it is not true that the attributes A and not-A cannot be applied to the same entity. Such paradoxes exist even at the foundations of mathematics. It is therefore necessary that an integrated approach to scientific inquiry be able to accommodate the actuality of this situation.

To see how this can be done, let us consider a domain of discourse, 'all meaningful statements'. Let us then represent the Law of Excluded Middle by two points with nothing in between:

We can then draw a line between the extreme points of the range to include the excluded middle, which represents statements whose truth or falsity we are doubtful about:



We now have a continuous domain of values for all meaningful statements, which is bounded by those statements that are certainly true or false. So we can say that those statements that are either true or false are in a set of certainties, which is the dual of the set containing those statements that are uncertain. In other words, the ends of the true-false spectrum of values can be considered to be the dual of the intermediate values. Any domain of values that consists of a range from one extreme to the other can be put into the set of all entities with this property.

Now as the limits of such a domain of values have the common property that they are extreme values, we can bend the line that represents the spectrum of values to form a circle so that the ends join. We can call this circle the Circle of Duality, another pattern that lies in the ontological level of the foundations.

Political systems provide a good example of the use of this tool of thought. Extreme left and right political systems, which are most commonly called communism and fascism, respectively, are both totalitarian forms of government, in contrast to more open systems that

favour the individual, such as liberalism. Socialism and conservatism would then be represented by the left and right sides of the circle of duality.[18]

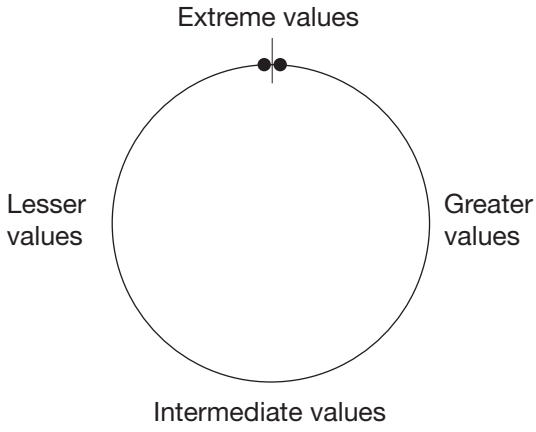


Figure 3.7: *The Circle of Duality*

I learned such a view of how we govern ourselves in the 1950s as a teenager, shortly after the end of the Second World War. Today, another perspective is emerging, highlighted by the American election in 2012. It is vitally important here to distinguish liberalism and individualism. Republicans, on the right of the political spectrum, want less government, attempting to preserve what they believe is individual freedom in the so-called free economy, where self-centred market forces are the only things that matter, believing that Democrats, as liberals, want more government.

But this is a gross misunderstanding. What liberals are seeking is to balance the needs of individuals and society, as illustrated by the call for bipartisanship. This is an intelligent, both-and approach to governance, following Jesus' call to love your neighbour as yourself, in contrast to the either-or approach of the Christian fundamentalists, seeking to defend their rigid religious beliefs, which give them a false sense of security and identity in the world.

This move towards bipartisan government will eventually lead to the end of politics, as we know it today. For political parties that represent opposite approaches to politics are, of necessity, dualistic. This can be seen most clearly from the root of the word *party*, which is Latin *partire*, meaning 'to divide'. So to be attached to one or other of these parties for one's own selfish needs, or even to support a political system consisting of parties, must inevitably lead to divisions in society.

However, *partire* has some other meanings: 'to share out' and 'to distribute'. With this radically different view of a party, we could, perhaps, begin to heal the divisions that exist in society today and begin to build the Sharing Economy. Indeed, we could all have a party to celebrate!

The triangle of duality

Having considered statements that are either true or false and those that are neither true nor false, we must now consider statements of the type 'This sentence is false'. How can we represent this in our analysis of all meaningful statements? Well, we can do this by turning to the foundations of mathematics.

When Georg Cantor developed his set theory in the 1870s, it was soon realized that this theory could serve as the base for all mathematics. However, he soon discovered paradoxes at the very heart of the theory, which the Western mind cannot allow. The problem lies with the notion of countable sets.

A set is countable if you can count its objects. Fairly obviously, a finite set is countable. An infinite set is also regarded as countable if all the elements in the set can be put into sequential order in one-to-one correspondence with the natural numbers: positive integers, which may also include zero. For instance, the even and odd integers and even all the rationals are countable. To see this, we simply use a mapping like this, where n is any integer: $2n \rightarrow n$.

Indeed, it is not difficult to prove that the number of rational numbers, of the form p/q, maps to the integers, even just those that lie between 0 and 1. For the rationals can be mapped to the integers as in Figure 3.8, as Cantor proved. Thus, we have this mapping: $1/1 \rightarrow 1$, $1/2 \rightarrow 2$, $2/1 \rightarrow 3$, $3/1 \rightarrow 4$, $2/2 \rightarrow 5$, $1/3 \rightarrow 6$, and so on. Yes, many rationals are duplicated, but this doesn't matter; it merely serves to emphasize the point. This mapping shows that the count of rationals is the same as the integers, even though there are an infinite number of rationals between any two integers. Surprisingly, the sets of integers, even integers, and rationals have the same level of infinity. This gives rise to some very strange arithmetic, like this: $\infty + \infty = \infty$.

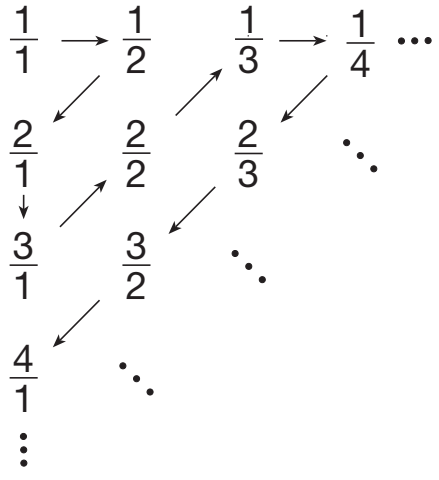


Figure 3.8: *Mapping of all rationals to countable integers*

However, Cantor showed that the real numbers, which include $\sqrt{2}$ and π, are not countable, denoted by the symbol $\mathfrak{c}$ for continuum. Even the number of real numbers between 0 and 1 is not countable. He used a diagonal argument that is described in Wikipedia, for instance, on the Internet.[19] So there are at least two levels of infinity. Indeed, Cantor went even further. He proved that there are actually an infinite number of transfinite cardinals, in two ways. He first made a distinction between cardinals and ordinals, the latter referring to ordered sets, the ordinal of the integers in increasing order being denoted by ω. He then showed that the cardinal number of all possible sets of ordinals with a countable ($\aleph_0$) number of members is larger than $\aleph_0$, which he denoted by $\aleph_1$. Continuing in this fashion, he defined the infinite series $\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_3$, and so on.

Cantor also defined an infinite series of transfinite cardinals through the notion of a power set, the set of all subsets of a set. For instance, the power set of {a b c} has eight members (2^3): {{a b c} {a b} {b c} {c a} {a} {b} {c} {}}. In general, the cardinality of the power set of any set

$\mathfrak{S}$ with n members is 2^n, which also applies to infinite sets. For Cantor proved that the power set of the set with $\aleph_0$ members has a higher cardinality than the set itself. So there is also an infinite series of such power sets, which mathematicians do not normally denote with signs, for some strange reason.

The obvious question now to ask is whether $\mathfrak{c} = \aleph_1$? In other words, does $\aleph_1$ equal $2^{\aleph_0}$? If so, "There is no set whose cardinality is strictly between that of the integers and that of the real numbers," as Cantor hypothesized in 1874.[20] Proving that the real number continuum is the smallest non-countable set was the first of 23 unresolved mathematical problems that David Hilbert posed at the International Congress of Mathematicians in Paris in 1900,[21] known as the continuum hypothesis. Applying this hypothesis to the two ways of creating an infinite series of infinite cardinals, the generalized continuum hypothesis asks whether it is true that $2^{\aleph_n} = \aleph_{n+1}$? In the event, Kurt Gödel in 1940[22] and Paul Cohen in 1963[23] showed that the hypothesis can neither be disproved nor be proved using the axioms of Zermelo-Fraenkel set theory, provided these axioms are consistent. Or we could say that the generalized continuum hypothesis is both unprovable and undisprovable through axiomatic, linear reasoning.

Nevertheless, for all practical purposes, it is reasonable to assume that the infinite series of power sets includes all distinct infinite cardinals, denoting the 'largest' infinity as $\aleph_\infty$, where ∞ is $\aleph_\infty$, defined recursively *ad infinitum*! But if people egoically believe that a separate, immortal soul either reincarnates indefinitely or has everlasting life, which infinity are they referring too?

The infinity of infinities led to the discovery of paradoxes at the very heart of mathematics. Let us define set A as the set of all sets, which must be the biggest set that can be defined. However, the power set of A is bigger than A, negating the premise. This led to a major crisis in the foundations of mathematics, for the concept of set, which is central to semantics, is more fundamental than that of number. To this day, this crisis has only been resolved by artificial means,[24] preventing us from looking at the Universe just as it is, without imposing any preconceptions.

Cesare Burali-Forti and Bertrand Russell subsequently found other paradoxes in set theory, which led to the famous 'crisis in the foundations' of mathematics.[25] For if set theory contains contradictions, it would be possible to prove any theorem and its opposite to be true, and the great edifice of mathematics would come crashing down. This situation is amusingly described in this little anecdote:

> The analyst G. H. Hardy once made this remark at dinner, and was asked by a sceptic to justify it: 'Given that 2 + 2 = 5, prove that McTaggart is the Pope'. Hardy thought briefly, and replied, 'We know that 2 +2 = 4, so that 5 = 4. Subtracting 3 we get 2 = 1. McTaggart and the Pope are two, hence McTaggart and the Pope are one.[26]

I have seen a similar story in another book where Bertrand Russell was the mathematician. So how can we incorporate Russell's paradox into IRL? Well Russell defined a set M as 'the set of all sets that do not contain themselves as an element'. Now is this set, M, a member of M or not? If it is a member, then M contains itself as a member, which is contradictory to the definition of M. But if M does not contain itself as a member, it should, by the definition of M, be included in M.

Let us see how we can represent such paradoxes in IRL. We can define dual sets A and B that are the sets of all sets that do and do not contain themselves as an entity, respectively. If we assume that in the domain of discourse that we are considering that we are certain whether a set is a member of itself or not, then the Law of Excluded Middle holds. All sets must therefore be either in A or B. So which sets are A and B in? It is fairly obvious that set A is a member of A. But what about set B? As B is the set of all sets that do not contain themselves as a member, B cannot be a member of B for then B would then contain itself as a member. But neither can B be a member of A. For then A would include a set that was not a member of itself. So B, like M, is a set that is in neither A nor B.

We are now faced with a crisis in the development of IRL. I have said that all concepts in IRL are formed in exactly the same way, for if they were not, the model, as a whole, would not be consistent. So entities with similar properties are put into one set, while those with different properties are put into different sets. But now we have two concepts in the model, uncertainties and paradoxes that apparently have the common property that they are both neither true nor false. How are we to distinguish them?

Well, I have said that by the Law of Excluded Middle, all sets in the domain being considered must be in either A or B. So if a set is not in B it must be in A. Similarly, if B is not in A it must be in B. So we can say that paradoxes are those entities that are both true and false at the same time. By this definition we can see that the Principle of Duality, which is both true and false, is a paradox. This can be seen even more clearly if we look at the more familiar self-referencing statement, "This sentence is false." For if that sentence is false, it asserts that it is true, and if it true, then it is false. So it is both true and false.

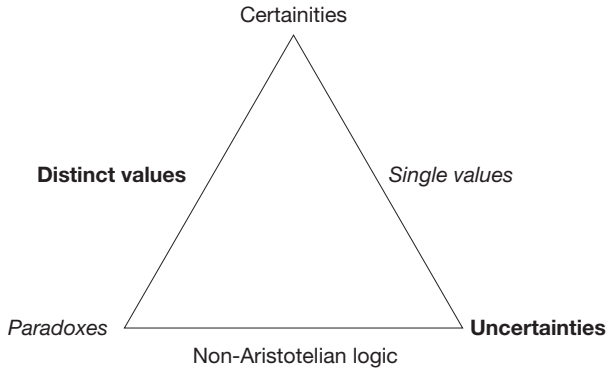


Figure 3.9: *The Triangle of Duality*

So, in conformity with the aim of the model to be complete and whole, we now have a means of classifying all meaningful statements into three all-inclusive categories:

1. **Certainties**, which are either true or false.

2. **Uncertainties**, which are neither true nor false.
3. **Paradoxes**, which are both true and false.

These three classes form a complete set that can be represented by the vertices of a triangle, in which each vertex is the dual of the other two in some sense. We can call this triangle the Triangle of Duality, another element in the ontological level of the foundations.

The cross of duality

There is one other geometric figure that well illustrates a common pattern with dualities, that is the cross. This figure arises when we consider two pairs of opposites, A and not-A and B and not-B. As this is such a common pattern, it is worthwhile to consider it as part of the data model in IRL. For this pattern exists across all disciplines, independent of interpretation.

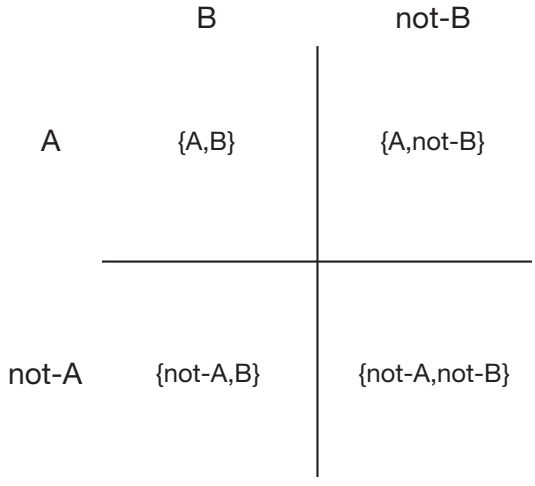


Figure 3.10: *The Cross of Duality*

The characteristics of entities with four possibilities of pairs of attributes can conveniently be placed inside each of the quadrants. There is no reason why this should be two-dimensional other than this is easily represented on a page or on a computer screen. In principle, we could divide a domain of discourse into any number of pairs of opposites, resulting in a multidimensional cross of duality.

We can illustrate the cross of duality with Jung's psychological types. Jung considered four functions that we use to relate to the world, including our inner world. These are divided into two pairs of opposites, thinking and feeling, which Jung considered rational, and sensation and intuition, the irrational functions.

So in this case, we have additional pairs of opposites. In general, the pairs {A, not-A} and {B, not-B} are not related to each other. But in Jung's theory of psychological types, these pairs are themselves opposites. To illustrate the rational functions, thinking and feeling types make decisions based on an objective examination of the facts and by attaching a subjective value to something, respectively. In contrast, sensation and intuition types lead lives predominately based on what reaches them through the senses and the unconscious, respectively, without attempting to rationalize these perceptions.

On top of these two pairs of opposites, Jung also considered another dimension, his well-known extrovert and introvert types. So we really need to draw a three-dimensional cross to illustrate all these characteristics. There are thus extrovert and introvert thinking, feeling, sensation, and intuition types. Of course, no one individual is wholly within one of these eight

categories. We all have characteristics in each of these octants. It is just that we have them in varying degrees as unique individuals.

This example illustrates a key point in IRL. We do not need to understand Jung's analysis in detail to see the underlying patterns in his reasoning. This approach to learning helps greatly whenever we are faced with a new domain. In conformity with the principle of simplicity, we can begin by looking at the simple patterns, which we can then use to build more complex structures as necessary.

This is especially necessary when attempting to read as complex a writer as Ken Wilber. Starting with *Sex, Ecology, Spirituality: The Spirit of Evolution*, published in 1995, Ken makes much use of the simplicity of the cross of duality in explaining his four-quadrant model of the Kosmos,[27] which was originally known as AQAL (all quadrants, all levels), but now embraces all quadrants, all levels, all lines of development, all states of consciousness, and all types of awareness, illustrated in Figure 1.50 on page 171.[28]

In this model, A and not-A are individual and collective, and B and not-B are interior and exterior. The key point here is that we need to look at all four of these quadrants if we are to develop an integral view of the Kosmos. Yet, because of our fragmented approach to learning in the past, few have yet been able to do this. Indeed, scholars in each of these quadrants have often been at war with those in the opposite quadrant, apparently unable to see all sides of the situation, such is the limitation of the dualistic mind.

The upper-left quadrant concerns the subjective realm of interior-individual, the world of I. The students of this realm are typically depth psychologists and spiritual teachers, such as Sigmund Freud and Gautama Buddha. The exterior-individual, objective world is studied by the behaviourists and mainstream scientists like B. F. Skinner, Stephen Hawkings, and Richard Dawkins. This is one half of the world of 'it'.

The second half of the it-world is the lower-right quadrant, the exterior-collective, which concerns social systems viewed empirically. Systems theorists, such as Ilya Prigogine and Fritjof Capra, and economic philosophers, like Karl Marx, typify the scholars in this realm.

And fourthly, there is the interior-collective or cultural realm in the lower-left corner of the cross of duality. For me, this is by far the most important of the four quadrants because this provides the cultural context in which we interpret our experiences and relate to each other in community. This is especially important at this time, with the approaching death of Western civilization and the global economy.

It is from this quadrant, then, that we shall make the scientific and cultural transformation that is so urgently needed in the world today. It is not surprising, therefore, to find that Ken puts Thomas Kuhn, the author of *The Structure of Scientific Revolutions* in this quadrant. Another I would put here is Erich Fromm, who sought to synthesize and deepen the two adjacent quadrants, most specifically the works of Freud, the Buddha, and Marx.

Dualism, duality, and Nonduality

To summarize this chapter on unifying opposites, we can look at the three principal ways in which we deal with opposites in our lives: dualism, duality, and Nonduality. The first two approaches correspond to contradiction and complementarity, respectively, and the last to unity, which we shall look at more in the next chapter.

Dualism is illustrated in Figure 3.11. This shows two opposites, *A* and *~A* (not-*A*) with a thick wall between them, where *A* is any being whatsoever, rather like the way *x* represents a number in mathematics. In dualism, there is thus a separation between opposites. In Western philosophy, dualism most commonly means mind-body dualism. But dualism is far more pervasive than this.

Most particularly, there is one dualism that is fundamental to Western civilization because it underlies all the others. This dualism arises because all the monotheistic religions regard God as other, for reasons that we explore in Subsection 'The birth of Christianity' in Chapter 11, 'The Evolution of the Mind' on page 859.

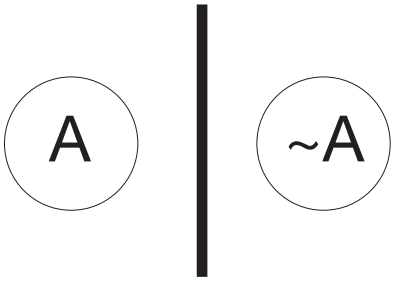


Figure 3.11: *Dualism*

Thankfully, the mystics of the monotheistic religions are beginning to speak out, to challenge the theological doctrines of the faiths that they belong to. For instance, the Benedictine monk David Steindl-Rast has said that one of his great concerns is that the Western God-view is warped and makes us sick. The idea of God as being separate from us is an extremely dangerous view.[29]

Now when God is seen as other, it is but a short step to seeing Nature as other, to be exploited and controlled for the ego-centred desires of humanity. That, in essence, is what happened to Western civilization following the scientific revolution of the sixteenth and seventeenth centuries, expressed most clearly by Francis Bacon when he said that the purpose of science is the "relief of man's estate".[30] Today, this dualistic mode of living is leading to ecological devastation, as Al Gore has made the world aware through his Oscar-winning documentary *An Inconvenient Truth*.

When Nature is other, it is but another short step to seeing our fellow human beings as other, likewise to be exploited and controlled to our bidding. It is, of course, the feeling of separation, of alienation, that leads us to fight and compete with other human beings for the precious resources of this beautiful planet of ours, a situation that is causing severe psychological distress in the world today.

What we tend to do in this dualistic mode of living is to identify with *A*, regarding *~A* as other, maybe even as an enemy, to be afraid of. We are thus unable to see the other person's

point of view, or in the case when *~A* is God, God's point of view. There is thus a tension or conflict between opposites; in dualism, opposites are regarded as being contradictory.

We can most clearly see our dualistic behaviour when countries go to war. When each country believes that God is on their side, they are unable to see the point of view of the people they regard as the enemy. An obvious example of this is the phrase, "God bless America," with which American presidents often end their speeches. Why not "God bless everybody"? Doesn't everyone on this planet deserve God's blessings, whatever they might be?

Yet there is an alternative to war, which we can illustrate with a well-known psychological test. An infant is first shown a card painted yellow on one side and blue on the other. Then the card is held in front of the infant so that she or he can see only the blue side, with the yellow side facing the tester. The tester then asks the infant, "What colour can I see?"

At six years of age, the infant generally answers 'blue'. He or she cannot see the other's perspective. Yet at about eight years of age, the answer is 'yellow'. The infant has grown into childhood.

So can Western civilization, in particular, and the human race, in general, grow out of infancy into childhood and thence into full maturity as a community of divine, liberated, conscious, loving beings?

Of course, it can. The first step in freeing ourselves of the egoic mind, thereby healing our troubled society, is to remove the barrier between the opposites, as in Figure 3.12. We then move from dualism to duality. Duality recognizes the fundamental fact of existence that opposites can never be separated; they are like the two sides of a coin. Contradiction has become complementarity.

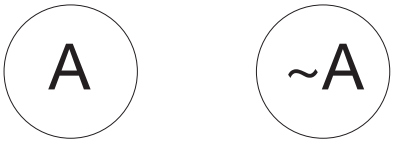


Figure 3.12: *Duality*

From the point of view of ourselves as individuals or groups, we can thereby see both our own and the other's perspective. It is in duality that compassion arises. A familiar example of this is John Gray's best-selling book *Men are from Mars, Women are from Venus*, helping women and men to understand their differences and so live more harmoniously together.

Not only this. It is widely recognized today that we human beings are not exclusively masculine or feminine. We all display characteristics of these opposite tendencies in some proportion or other. Psychologists recognize many other opposites, both of which are present within us, of which Carl Jung's concepts of extrovert and introvert are perhaps the best known.

Now the key point here is that these examples show that Aristotle's Laws of Contradiction and Excluded Middle, which are fundamental to mathematical proof and logical inference, are not universally true. The fact that the Law of Contradiction is not universally true led the physicists to great consternation in the first half of the twentieth century. For, as is well

known, they discovered that light, in particular, and electromagnetic radiation, in general, display contradictory properties. Sometimes light behaves as a particle, existing in a small region of space, and sometimes it behaves like a wave, spread out in space.

To overcome what looked like an absurdity, Neils Bohr introduced the notion of 'complementarity'. As Fritjof Capra tells us, "He considered the particle picture and the wave picture two complementary descriptions of the same reality, each of them only partly correct and having a limited range of application".[31]

But there is an even more fundamental contradiction in physics, which was partially resolved by David Bohm with his theory of the implicate order. These opposites of the implicate and explicate orders provide us with a clue about how we can develop the theory of everything, which reconciles all opposites, not just some. So far, we have mainly been considering the relativistic world of form. But this also has an opposite: the formless Absolute Whole. Now it is impossible to escape opposites in the world of form; it is, by its very nature, dual. On the other hand, the Absolute is Nondual, it transcends all opposites.

So to return Home to Wholeness, we need to unify Nonduality and duality, which we shall do in the next chapter. This third situation is illustrated in Figure 3.13, showing Nondual, limitless Consciousness embracing all opposites, including Nonduality and duality and science and spirituality. Two has become one, the unity in diversity.

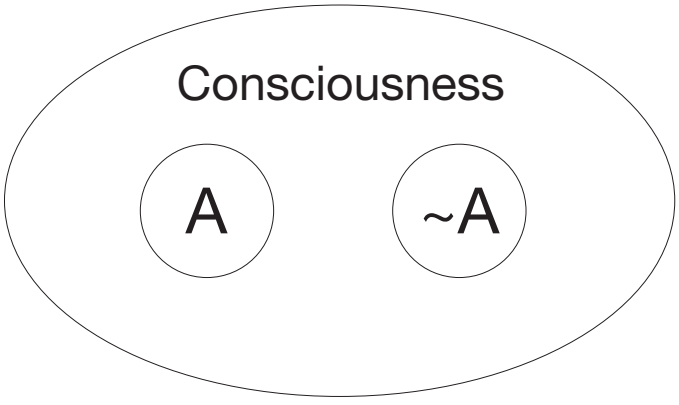


Figure 3.13: *Nondual Consciousness*